We will describe and discuss four classes of photoionization matrix element zeros and their physical consequences. Three of these classes have been discussed in previous publications and are given as follows: i) low energy zeros that result in Cooper minima (CM) (Fano and Cooper, 1968; Msezane and Manson, 1975, 1982; and Yin, 1987a) (nonrelativistic and relativistic); ii) point Coulomb relativistic zeros (PCRZ), that only occur above threshold when $Z \geq 128$ (Yin and Pratt, 1987b); iii) higher energy nonrelativistic Coulomb zeros (HENRCZ) that only occur in beyond dipole matrix elements at photon energies E^p between 1 keV and 50 keV (Wang, et. al, 1982). The fourth class of zeros, which were discovered more recently, occur at photon energies above 100 keV (for $l_b < 6$, l_b is bound state quantum number). The occurrence and positions of these zeros is governed by very simple rules, given below. In this report we will compare this newer class of zeros, which we will call relativistic high energy zeros (RHEZ), with the other three classes of zeros.

Dipole RHEZ were first discovered by Y. S. Kim (private communication), then discussed by (Tong, et al, 1994) as occurring at energies of the order mc^2. They were characterized as being independent of n and Z and as having a simple dependence on l_b with regard to photon energies at which they occur (LaJohn and Pratt, 1999). The dipole RHEZ only occur in $(n, l_b, j_b) \to (\epsilon, l_c = l_b + 1, j_c = j_b)$ transitions ($|\kappa_b| = |\kappa_c|$ where $\kappa_b < 0$ and $\kappa_c > 0$, subscripts b and c refer to bound and continuum states respectively); their positions (though not the magnitudes of the matrix elements) are completely independent of the central potential as well as retardation.

In the dipole case RHEZ occur at photon energy positions $E^p_{l_b} = mc^2/(l_b + 1)$. These results, first obtained numerically, can be demonstrated to be an exact consequence of the Dirac equation. Higher multipole RHEZ do exist and appear very similar in behavior, however their photon energy positions range from nearly independent to somewhat dependent on n and Z. Simple but approximate empirical formulas for their positions have been obtained.

Although RHEZ occur at very high energies, they do have physical consequences. The most striking effects occur in the photon-electron polarization correlation coefficient C_{23}. C_{23} is the coefficient that corresponds to the correlation of longitudinally polarized electrons with linearly polarized photons (in atomic photoionization). Although C_{23} vanishes in the nonrelativistic limit, it is quite sizable at intermediate to large Z and at intermediate energies (Pratt, 1964). A sign change in C_{23} occurs at photon energies near the position of the dipole RHEZ at ejection angles less than about 50^o. One reason why the effects due to RHEZ are greatest in C_{23} is that it is the only C_{ij} that is composed of only interference terms, all products of reduced matrix elements of the same multipole are zero.

Low energy zeros that result in Cooper minima (CM) occur at near threshold energies, in the case of ground state nonrelativistic dipole matrix elements only in $(n, l_b) \rightarrow (\epsilon, l_c = l_b + 1)$ transitions in relativistic $(n, l_b, j_b) \rightarrow (\epsilon, l_c, j_c)$ transitions, where $l_c = l_b + 1$ with $j_b = l_b - 1/2$ and $j_c = l + b + 1/2$ with $j_b = l_b + 1/2$ and $j_c = l_b + 1/2$ with $j_b = l_b + 1/2$ and $j_c = l_b + 3/2$. Simple rules pertaining to the number of zeros as well as whether the number of zeros is even or odd have been established (Pratt, et al, (1987), Oh and Pratt, (1996)). Unlike RHEZ, low energy CM zeros are very dependent on the electronic properties of an atom (i. e. electron charge density distribution, wavefunction overlap and nodes, as well as screening). This is reflected in a strong dependence of their positions on n, l and Z. Such zeros display splitting between j states (of a given l) that can be larger than that that which can be attributed to spin orbit splitting of bound energy levels. This effect has been linked to the presence of a centrifugal barrier (Yin and Pratt, 1987a). Low energy CM zeros result in major physical consequences. Examples are minima in total cross sections and sign changes in asymmetry parameters.

Point Coulomb relativistic zeros (PCRZ) occur in dipole transitions. At above threshold energies they occur in $ns_{1/2} \rightarrow \epsilon p_{3/2}$ transitions when $Z \geq 128$ and in $np_{1/2} \rightarrow \epsilon d_{3/2}$ transitions when $Z \geq 133$. Unlike RHEZ, PCRZ occur at near threshold energies. These two classes of zeros do not occur for the same transitions. In PCRZ, the number of zeros is even when $|\kappa_b| = |\kappa_c|$ $(l_c = l_b + 1)$ and is odd when $|\kappa_c| - |\kappa_b| = 1$.

Higher energy nonrelativistic Coulomb zeros (HENRCZ) occur at energies ranging from 1-50 keV (energies greater than where CM occur but less than that for RHEZ). They occur in beyond dipole matrix elements. In the quadrupole case, they only occur in $(n, l_b, j_b) \rightarrow (\epsilon, l_c, j_c)$ transitions in which $l_c = l_b + 1$ and $j_c = l_b + 3/2$. Unlike RHEZ, the positions of HENRCZ are dependent on n and Z. The are also largely independent of electron screening. Also, like low energy CM, strongly affect angular distributions.

In conclusion, RHEZ differ from all other photoionization matrix element terms in the following ways. In dipole matrix elements, the position of the zero is completely independent of n, Z, V (the central potential) and retardation and simply dependent on l ($E^p_{l_b} = mc^2/l_b + 1$). However higher multipole RHEZ show some dependence on n and Z and a somewhat more complicated dependence on l. Despite the fact that RHEZ occur at energies on the order of mc^2, they still have marked physical consequences with respect to photon-electron polarization correlations, especially C_{23}.

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