

Entanglement Change in Atomic Photoionization

Young Soon Kim^{1,*}, Yu Jin Kim¹ and R. H. Pratt²

¹Department of Physics, Myongji University, Yong-In 449–728, Korea

²Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, U.S.A.

PACS Ref: 32.80.Fb, 03.67.Mn

Received August 29, 2003; accepted November 17, 2003

Abstract

Photoionization is one of the basic processes of light-matter interaction, which may be used to transfer at least part of quantum information from photons to electrons in the continuum in such a way that the final state electron carries composite information of the initial electron and of the incident photon. We examine how photoionization may change entanglement of electrons. Our result shows that entanglement in one degree of freedom is affected by a transition in another degree of freedom through the coupling of the two degrees of freedom.

1. Introduction

Entanglement has been studied intensively in recent years in the context of quantum information or quantum computation. It is the core idea of quantum nonlocality, in the sense that a measurement on one particle from the entangled pair will instantly determine the outcome of the corresponding property of the entangled partner, which may have moved far away from the first particle, and that the outcome may at times violate some of the obvious classical predictions. The typical EPR pairs in one of the four Bell states are by now well-known examples of bipartite entanglement. Suggestions are being made for utilizing such properties in quantum communication or quantum cryptography, as well as in quantum computation [1].

Entanglement has been discussed, for example, for the cases of pairs of photons or of electrons, or of atoms. Entangled photon pairs are typically generated via parametric down conversion. Generation of entangled electron pairs through absorption of a single photon, namely in double photoionization of atoms, where electron correlation plays an essential role, has been discussed [2]. However, this process usually involves very small cross sections and high energy photons. Generation of entangled atoms in a cavity [3] or trapped in separate cavities [4], identified by detection of emitted polarized photons, has been suggested for the case of two Λ -type three-level atoms.

We have been studying entanglement in atomic processes, utilizing more realistic description of atoms, for the cases of atomic photoionization [5] and of electron-divalent atom scattering [6]. Here, we wish to discuss whether and how entanglement changes as a result of an atomic process, in various situations in which one of the bound electrons is sent away through absorption of one photon; photoionization of atoms. Our formalism mostly utilizes the framework of independent particle approximation (IPA), with which we can present our essential idea while unnecessary complications are avoided. We hope that this work may

provide a useful indication of methods for recording, storing and retrieving quantum information.

2. Formalism

2.1. Spin states of photoelectrons ejected from an initial ns state

We study cases of one or two initial ns electrons in one or more atoms, in single configurations within an IPA model. These ns states are frequently encountered in ground states of one- and two-valence electron atoms. The final continuum states of energy ε reached via absorption of one photon, assuming that electric dipole approximation is valid, are $\varepsilon p_{1/2}$ and $\varepsilon p_{3/2}$. Let us denote the two radial dipole matrix elements as R_1 and R_3 , respectively. Note that $R_1 = R_3$ if spin-orbit interaction in the final continuum states is neglected. (We will elsewhere discuss the case of $l > 0$ initial orbital angular momentum states where spin-orbit interaction plays a less important role. In our previous study on correlations between incident photon polarization and photoelectron spin polarization, the photoelectrons ejected from a definite initial (total) angular momentum state with $l > 0$ could have a large degree of spin polarization even with negligible spin-orbit interaction [7].)

When the incident photon is right-circularly (σ^+) polarized, the final photoelectron spinor is given [8] as

$$-\sqrt{2/3} \begin{pmatrix} R_3 Y_{11} \\ 0 \end{pmatrix} \equiv \begin{pmatrix} a \\ b \end{pmatrix}$$

when the initial electron spin is up, $|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and as

$$-1/3\sqrt{2/3} \begin{pmatrix} \sqrt{2}(R_3 - R_1)Y_{10} \\ (2R_1 + R_3)Y_{11} \end{pmatrix} \equiv \begin{pmatrix} c \\ d \end{pmatrix}$$

when the initial electron spin is down, $|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The quantization axis is given by the direction of light propagation. (In these expressions, multiplication of the appropriate phase factor $e^{i\delta_j}$ to each of the radial dipole matrix elements is understood and not explicitly shown here for simplicity of notation [9].) If the incident photon is left-circularly (σ^-) polarized, we obtain the final spinor $\begin{pmatrix} -d \\ -c \end{pmatrix}$ when the initial electron spin is up, and $\begin{pmatrix} -b \\ -a \end{pmatrix}$ when the initial electron spin is down.

The process of photoionization from an ns electron with a σ^+ photon may be considered as characterized by an operator of the form $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$, and by $\begin{pmatrix} -d & -b \\ -c & -a \end{pmatrix}$ with a σ^- photon. These operators are not unitary and the process is

* Associate member of Korea Institute for Advanced Studies. e-mail: yskim@mju.ac.kr

probabilistic; we cannot call these operators single-qubit quantum “gates” although they formally act on an arbitrary input qubit state to produce a well-defined final qubit state once the electron is ionized. We concentrate on the cases where the photoionization process has occurred, and on a time scale short enough to neglect any possible decoherence mechanism due to other interactions. We have angle- and energy-dependent operators which partially translate the photon polarization into spin states of photoelectrons ejected from an initial ns state with arbitrary spin. When the two transition amplitudes are equal, which corresponds to neglecting spin-orbit interaction in the final continuum, then in this case, $d = a$ and $c = 0 = b$, so that the spin state of the electron does not change through photoionization, and the operators reduce to the identity.

2.2. Concurrence as a measure of entanglement

In this work we are interested in the entanglement of the spins, rather than the spin polarization itself, of the electrons. Among several widely accepted measures of the degree of entanglement for bipartite systems [10], which are all related, we will here choose the quantity called concurrence as the measure of entanglement, due to its simplicity, especially for pure bipartite states, to which we will mainly restrict our attention.

For a two-qubit system in a normalized pure state denoted by

$$|\Phi\rangle = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \equiv \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix}, \quad (1)$$

the concurrence is simply given by [9]

$$C(|\Phi\rangle) = 2|\alpha\delta - \beta\gamma|, \quad (2)$$

where we write, for example,

$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle/\sqrt{2} \equiv 1/\sqrt{2} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right] \equiv 1/\sqrt{2} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}.$$

Note that $C(|\Phi\rangle) = 0$ if and only if $|\Phi\rangle$ is a product state, and $C = 1$ for maximally entangled states such as the Bell states:

$$|\Psi^\pm\rangle = 1/\sqrt{2}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle), \quad |\Phi^\pm\rangle = 1/\sqrt{2}(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle).$$

3. Entanglement change in photoionization of atoms

3.1. Photoionization of an initial $ns^2\ ^1S$ state with σ^+ photon absorption

Suppose we have a divalent atom in its ground state $ns^2\ ^1S$, which is the most common spin-entangled state supplied by nature,

$$|ns^2\ ^1S\rangle = \psi_{ns}(\mathbf{r}_1)\psi_{ns}(\mathbf{r}_2) \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}, \quad (3)$$

which is in fact in one of the Bell states denoted as $|\Psi^-\rangle$. Clearly the concurrence for this state is 1.

We know that when spin-orbit interaction is negligible the LS -coupling scheme is valid and singlet-triplet transitions (leading to intercombination lines) are forbidden. This may be viewed as a natural consequence of the last remark of Section 2.1. In such cases the ejected photoelectron and the remaining bound electron of the ion still form a singlet, and the final state is 1P . The two electron spins are entangled in exactly the same manner as before.

However, spin-orbit interaction becomes significant as the atomic number grows, and hence the final state may not be described as a pure singlet state. Naturally, one may expect that there will be a change induced in the entanglement of the two-spin system. Note that photoabsorption itself is spin-independent and induces the transition in the spatial part of the wave function, and that the spin and orbital angular momentum coupling to form the total angular momentum state induces the changes in the electron spin state.

With σ^+ photon absorption, our two-qubit operators may be expressed as

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} a & c \\ b & d \end{pmatrix},$$

and the final spin state as

$$\begin{aligned} & \frac{1}{\sqrt{2}} \left[-\sqrt{\frac{2}{3}} \begin{pmatrix} R_3 Y_{11} \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \left(-\frac{1}{3}\right) \sqrt{\frac{2}{3}} \begin{pmatrix} \sqrt{2}(R_3 - R_1) Y_{10} \\ (2R_1 + R_3) Y_{11} \end{pmatrix} \right. \\ & \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \left(-\frac{1}{3}\right) \sqrt{\frac{2}{3}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} \sqrt{2}(R_3 - R_1) Y_{10} \\ (2R_1 + R_3) Y_{11} \end{pmatrix} \\ & \left. - \left(-\sqrt{\frac{2}{3}}\right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} R_3 Y_{11} \\ 0 \end{pmatrix} \right] \\ & = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a \\ b \end{pmatrix} \right] = \frac{a+d}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \quad (4) \end{aligned}$$

yielding a final state concurrence unchanged from 1, even with non-vanishing spin-orbit interaction and hence with singlet-triplet mixing allowed, contrary to our intuition. The mixture is fully entangled in this case.

3.2. Photoionization of initial $nsms$ 1S or 3S state with σ^+ photon absorption

We next examine concurrence for the cases with initial states in $nsms$ 1S or 3S . The two-qubit operator may be written now as

$$\begin{aligned} & \left[\left\{ \begin{pmatrix} a & c \\ b & d \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} a & c \\ b & d \end{pmatrix} \right\} \frac{\delta_{mn} + 1}{2} \right. \\ & + \left. \left\{ \begin{pmatrix} a' & c' \\ b' & d' \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} a' & c' \\ b' & d' \end{pmatrix} \right\} \frac{(1 - \delta_{mn})}{2} \right] \\ & \times (2 - \delta_{mn}) \end{aligned}$$

with the primed quantities denoting the transitions from an ms state. When the initial state is given by

$$\begin{aligned} |nsms \ ^1S\rangle &= \frac{1}{\sqrt{2}} [\psi_{ns}(\mathbf{r}_1)\psi_{ms}(\mathbf{r}_2) + \psi_{ms}(\mathbf{r}_1)\psi_{ns}(\mathbf{r}_2)] \\ & \times \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}, \end{aligned} \quad (5)$$

the final spinor is written as

$$\begin{aligned} & \frac{1}{2} \left[-\sqrt{\frac{2}{3}} \begin{pmatrix} R_3 Y_{11} \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \left(-\frac{1}{3}\right) \sqrt{\frac{2}{3}} \begin{pmatrix} \sqrt{2}(R_3 - R_1) Y_{10} \\ (2R_1 + R_3) Y_{11} \end{pmatrix} \right. \\ & \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \left(-\sqrt{\frac{2}{3}}\right) \begin{pmatrix} R_3 Y_{11} \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \left(-\frac{1}{3}\right) \sqrt{\frac{2}{3}} \\ & \times \begin{pmatrix} \sqrt{2}(R'_3 - R'_1) Y_{10} \\ (2R'_1 + R'_3) Y_{11} \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \left(-\frac{1}{3}\right) \sqrt{\frac{2}{3}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ & \otimes \begin{pmatrix} \sqrt{2}(R'_3 - R'_1) Y_{10} \\ (2R'_1 + R'_3) Y_{11} \end{pmatrix} - \left(-\sqrt{\frac{2}{3}}\right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} R'_3 Y_{11} \\ 0 \end{pmatrix} \\ & + \left(-\frac{1}{3}\right) \sqrt{\frac{2}{3}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} \sqrt{2}(R_3 - R_1) Y_{10} \\ (2R_1 + R_3) Y_{11} \end{pmatrix} - \left(-\sqrt{\frac{2}{3}}\right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ & \otimes \begin{pmatrix} R_3 Y_{11} \\ 0 \end{pmatrix} \left. \right] = \frac{1}{2} \left[\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} c \\ d \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right. \\ & + \begin{pmatrix} a' \\ b' \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} c' \\ d' \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} c' \\ d' \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ & \otimes \begin{pmatrix} a' \\ b' \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a \\ b \end{pmatrix} \left. \right] \\ & = \frac{a + d + a' + d'}{2} \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}. \end{aligned} \quad (6)$$

The concurrence of the final state is again unchanged from 1.

The results for the initial triplet states

$$\begin{aligned} & |nsms \ ^3S; M_s = -1\rangle \\ & = \frac{1}{\sqrt{2}} [\psi_{ns}(\mathbf{r}_1)\psi_{ms}(\mathbf{r}_2) - \psi_{ms}(\mathbf{r}_1)\psi_{ns}(\mathbf{r}_2)] |\downarrow\downarrow\rangle, \end{aligned} \quad (7a)$$

$$\begin{aligned} & |nsms \ ^3S; M_s = 0\rangle \\ & = \frac{1}{\sqrt{2}} [\psi_{ns}(\mathbf{r}_1)\psi_{ms}(\mathbf{r}_2) - \psi_{ms}(\mathbf{r}_1)\psi_{ns}(\mathbf{r}_2)] \\ & \times \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}, \end{aligned} \quad (7b)$$

$$\begin{aligned} & |nsms \ ^3S; M_s = +1\rangle \\ & = \frac{1}{\sqrt{2}} [\psi_{ns}(\mathbf{r}_1)\psi_{ms}(\mathbf{r}_2) - \psi_{ms}(\mathbf{r}_1)\psi_{ns}(\mathbf{r}_2)] |\uparrow\uparrow\rangle, \end{aligned} \quad (7c)$$

are also surprising because the product states ($C = 0$) in spin degree of freedom, Eq. (7a) yield the final spin state concurrence 1.

The results obtained for various initial states are summarized in the following table, together with the results for the Bell states as initial states, assuming the initial spatial wave function for the two electrons is the product of the same states: $|\psi_{ns}\rangle|\psi_{ns}\rangle$, as in Eq. (3). This is possible for $|\Phi^\pm\rangle$ or for $|\Psi^\pm\rangle$ only if the two initially entangled electrons belong to two different atoms sufficiently far away, so as to neglect overlap of the spatial wave functions, due to the Pauli exclusion principle.

We can also examine the cases where the two electrons belong to two different atoms, with the spatial wave function $|\psi_{ns}\rangle|\psi_{ns}\rangle$, and the spins are in a product state given by either $|\uparrow\uparrow\rangle$ or $|\downarrow\downarrow\rangle$, with initial state concurrence 0. With a similar method, we obtain the final state concurrence 0 or $|c^2|/(|c|^2 + 2|d|^2)$, respectively.

4. Discussion

We have presented a way to estimate entanglement changes in photoionization within an IPA scheme. With this method we have also examined various other cases, such as two atoms in initial ns states with absorption of a polarization-entangled photon pair, symbolically described as $1/\sqrt{2}(\sigma^+\sigma^- + \sigma^-\sigma^-)$, where the two initial electrons either belong to the two different atoms in a product state given by $\begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} \otimes \begin{pmatrix} \alpha_2 \\ \beta_2 \end{pmatrix}$, or in an entangled state, with spatial wave functions entangled or unentangled, for example.

In the cases we have studied, the results for $|\Psi^-\rangle$ type of the initial spin state suggest a possibility of utilizing one photoelectron ionized from a pair of electrons initially in a singlet state as a flying qubit. But in other cases the entanglement change may occur as a result of photoionization, and the changes in general depend on the spatial configuration. This indicates that entanglement in one degree of freedom is affected by a transition occurring in another degree of freedom, through the coupling of the two degrees of freedom in the process. One has to be cautious in designing quantum devices utilizing entanglement proper-

Table I. *Initial and final state concurrence in photoionization with a σ^+ photon from different initial states.*

Initial two-electron state	Initial state concurrence	Final state concurrence
$ ns^2\ ^1S\rangle = \psi_{ns}(\mathbf{r}_1)\psi_{ms}(\mathbf{r}_2)(\uparrow\downarrow\rangle - \downarrow\uparrow\rangle)/\sqrt{2}$	1	1
$ nsms\ ^1S\rangle = 1/\sqrt{2}[\psi_{ms}(\mathbf{r}_1)\psi_{ms}(\mathbf{r}_2) + \psi_{ms}(\mathbf{r}_1)\psi_{ns}(\mathbf{r}_2)](\uparrow\downarrow\rangle - \downarrow\uparrow\rangle)/\sqrt{2}$	1	1
$ nsms\ ^3S; M_s = -1\rangle = 1/\sqrt{2}[\psi_{ns}(\mathbf{r}_1)\psi_{ms}(\mathbf{r}_2) - \psi_{ms}(\mathbf{r}_1)\psi_{ns}(\mathbf{r}_2)] \downarrow\downarrow\rangle$	0	1
$ nsms\ ^3S; M_s = 0\rangle = 1/\sqrt{2}[\psi_{ns}(\mathbf{r}_1)\psi_{ms}(\mathbf{r}_2) - \psi_{ms}(\mathbf{r}_1)\psi_{ns}(\mathbf{r}_2)](\uparrow\downarrow\rangle + \downarrow\uparrow\rangle)/\sqrt{2}$	1	1
$ nsms\ ^3S; M_s = +1\rangle = 1/\sqrt{2}[\psi_{ns}(\mathbf{r}_1)\psi_{ms}(\mathbf{r}_2) - \psi_{ms}(\mathbf{r}_1)\psi_{ns}(\mathbf{r}_2)] \uparrow\uparrow\rangle$	0	0
$ \Psi^+\rangle = 1/\sqrt{2}(\uparrow\downarrow\rangle + \downarrow\uparrow\rangle)$ with $ \psi_{ns}\rangle \psi_{ns}\rangle$ as the spatial part	1	$ a + d ^2/(2 c ^2 + a + d ^2)$
$ \Psi^-\rangle = 1/\sqrt{2}(\uparrow\downarrow\rangle - \downarrow\uparrow\rangle)$ with $ \psi_{ns}\rangle \psi_{ns}\rangle$ as the spatial part	1	1
$ \Phi^+\rangle = 1/\sqrt{2}(\uparrow\uparrow\rangle + \downarrow\downarrow\rangle)$ with $ \psi_{ns}\rangle \psi_{ns}\rangle$ as the spatial part	1	$ c^2 + 4ad /(2 a ^2 + c ^2 + 2 d ^2)$
$ \Phi^-\rangle = 1/\sqrt{2}(\uparrow\uparrow\rangle - \downarrow\downarrow\rangle)$ with $ \psi_{ns}\rangle \psi_{ns}\rangle$ as the spatial part	1	$ c^2 - 4ad /(2 a ^2 + c ^2 + 2 d ^2)$

ties, because interaction will change these properties in nontrivial ways.

Acknowledgement

We wish to thank D. Uskov, S. C. Oh, S. Lee, J. Kim, and S. D. Oh for helpful discussions. This work was supported by Korea Research Foundation Grant (KRF-2002-070-C00029). R. H. Pratt acknowledges support in part by NSF grant 0201595.

References

1. For example, "Quantum Computation and Information," (M. A. Nielsen and I. L. Chuang), (Cambridge University Press 2000); C. H. Bennett, et al., Phys. Rev. Lett. **70**, 1895 (1993).
2. Chandra, N. and Chakraborty, M., J. Phys. B: At. Mol. Opt. Phys. **35**, 2219 (2002).
3. Jongcheol Hong and Hai-Woong Lee, Phys. Rev. Lett. **89**, 237901 (2002).
4. Xun-Li Feng, Zhi-Ming Zhang, Xiang-Dong Li, Shang-Qing Gong, and Zhi-Zhan Xu, Phys. Rev. Lett. **90**, 217902 (2003).
5. Young Soon Kim and R. H. Pratt, ICPEACXX ('97. 7. 23. – 29., Vienna, Austria), Poster MO 011 (Vol. II) (1997).
6. Young Soon Kim and Yu Jin Kim, The 5th Asian International Seminar on Atomic and Molecular Physics (AISAMP5; 2002. 10. 2. – 10. 5., Nara, Japan) (2002), page 41.
7. Young Soon Kim, Goldberg, I. B. and Pratt, R. H., Phys. Rev. A **51**, 424 (1995).
8. Kessler, J. "Polarized Electrons," 2nd ed. (Springer-Verlag, Berlin, 1985).
9. Fano, U., Phys. Rev. **178**, 131 (1969); *ibid.* **184**, 250 (1969).
10. Wooters, W. K., Quantum Information Comput. **1**, 27 (2001).