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Radiation Physics and Chemistry 75 (2006) 624–630

Radiation Physics and Chemistry

www.elsevier.com/locate/radphyschem

Tseng's calculations of low energy pair production

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Abstract

We review the theoretical work 1971–1997 of H.K. Tseng on low energy pair production. In this work numerical calculations were performed in independent particle approximation in a screened self-consistent central potential, expanding the *S*-matrix element in partial waves and multipoles. Sampling techniques in partial waves and multipoles were used to extend the calculations to higher energies (up to 10 Mev). Total cross sections, the positron energy spectrum, the positron angular distributions, and the positron–photon polarization correlations were studied. Agreement was obtained with most experiments, although some anomalies remained at the lowest energies (particularly at 1082 keV). Atomic screening of the nuclear charge decreases cross sections at higher energies, as described by a form factor in the momentum transfer to the nucleus. In an intermediate energy regime point Coulomb results in a shifted energy spectrum may be used. At low energies screening increases cross sections, and this is characterized in terms of a normalization screening factor which describes the change in magnitude of electron and positron wave functions at small distances. In this low energy regime angular distribution shapes and polarization correlations are independent of screening.

Keywords: Pair production

1. Introduction

In this Special Issue devoted to pair production it is appropriate to review the work of H.K. Tseng. His papers, beginning in 1971, and the code on which they are based, still give us our main quantitative knowledge of the pair production process for energies less than a few MeV. These results include predictions for total cross sections, spectra, angular distributions, and polarization correlations. Tseng's initial work, utilizing his earlier work on the formally related process of bremsstrahlung, began while he was a graduate student at the University of Pittsburgh. The work was continued after he returned to Taiwan, to National Central University, Chung-Li.

An important focus in Tseng's work has been the characterization of atomic screening of the nuclear charge field in which the electron/positron pair is produced: how screening affects the process, and how the effect of screening changes with energy. For high energy photons a Born approximation theory is qualitatively correct, and it says that screening effects are characterized by an atomic form factor (decreasing the cross section), and can be important. At low energies momentum transfers to the nucleus must be big, small distances are important, and there is no form factor screening. But while at small distances wave function shapes are Coulombic, their magnitudes (normalizations) are changed by screening, leading to a normalization theory of screening (which can increase the cross

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0969-806X/$ - see front matter © 2005 Published by Elsevier Ltd.
doi:10.1016/j.radphyschem.2005.09.006

section). And that can be further improved in the transitional region by noting that then wave functions at relevant distances are actually Coulombic for a shifted energy.

In the next section we shall review the knowledge of pair production prior to Tseng's work. In Section 3 we shall review Tseng's Pittsburgh papers 1971–1974 and the continuing collaboration 1980–1981. In Section 4 we shall discuss Tseng's work 1990–1997 in Chung-Li. In Section 5 we discuss some other more recent work on low energy pair production. And finally in Section 6 we discuss some current questions and opportunities.

2. Previous work

Much of the older work on pair production, both theory and experiment, was reviewed by Motz et al. (1969).

There are relativistic Born calculations of Bethe and Heitler (1934), valid for low Z and high energies of all particles, leading in the nuclear point Coulomb potential to the Bethe–Heitler formula. When the nucleus is screened this Born result is modified by multiplication by a form factor, a Fourier transform of the charge density.

Note Born approximation is not valid when either (or both) the electron and positron are of low energy.

But even for high energies Born approximation is not valid for high Z. (This is to be contrasted with non-relativistic theories.) In the Coulomb case Bethe and Maximon (1954) obtained the high energy limit for all Z, using Sommerfeld–Maue wave functions, and the screening modifications of the high energy limit were also later obtained. [We note that in a very recent paper Lee et al. (2004) have obtained 1/energy corrections to this high energy result in the Coulomb case, which they found useful as low as 10 MeV.]

In the point Coulomb case there were numerical calculations in partial waves, valid for any Z, utilizing analytic properties of Coulomb wave functions. A few pioneering calculations were reported by Jaeger and Hulme (1936) and by Jaeger (1936, also subsequently). More extensive work was then reported by Overbo et al. (1968). [Sud et al. (1979) later reformulated these methods, so that Sud and coworkers subsequently were able to extend calculations to higher energies (like 20 MeV), toward the region of the Bethe–Maximon approach.]

There were significant discrepancies between these point Coulomb calculations and the few extant low energy experimental results. The question was whether this disagreement between theory and experiment could be explained by the neglect of screening effects in the theory. The Born approximation form factor said that screening would not be important at such energies, due to the large required momentum transfer to the nucleus (with a photon of more than an MeV impinging, and low energy low momentum electron and positron emerging). But Born approximation was not valid for low energy particles.

This was the question which Tseng's work addressed.

3. Tseng's approach, and work 1971–1974 and 1980–1981

The initial work was presented in three papers, Tseng and Pratt (1971, 1972, 1974). The primary focus was on the (positron) spectrum and the total cross section, with some discussion of angular distributions and later of polarization correlations. The photon energy range from 1.073 to 2.615 MeV was studied, and for the spectrum on up to 5 MeV.

The approach was to calculate the relevant relativistic S-matrix element, in independent particle approximation in a numerically given screened central potential, making a multipole expansion for the photon and a partial wave expansion for the electron and positron. Note that this neglects correlation and exchange, excitation and ionization of the target (except in a sum rule approximation). It truncates the series for multipoles and partial waves once they are believed convergent (which limits the energy range for which the calculation is valid). The calculations are subject to the usual demands on numerical accuracy.

Examples of the calculated positron spectrum obtained by Tseng and Pratt (1971) are shown in Figs. 1 and 2. Note that, contrary to Born approximation, the spectrum is not symmetric. We understand this because, at low energies, it is the small distance region which will be important, due to the required large momentum transfer to the nucleus. But a low energy positron wave function is suppressed near the nucleus.

However the shapes of the electron and positron wave functions at small distances are not much affected by screening, only their normalization. This explains why the shapes of angular distributions were found to be independent of screening, indicating that screening at these energies was simply a normalization effect. Contrary to Born approximation, this means that screening is important at low energy, because screening changes the magnitude of a low energy wave function. Screening can increase the cross section, as shown in the figures, because the magnitude of a low energy positron wave function at small distances is larger in a screened potential than in an unscreened Coulomb potential.

More data on the positron spectrum from Tseng and Pratt (1972) is shown in Figs. 3, 4 and 5, illustrating the approach to the symmetric Born form with increasing energy. At these higher energies, instead of using a normalization screening theory, it was possible to relate

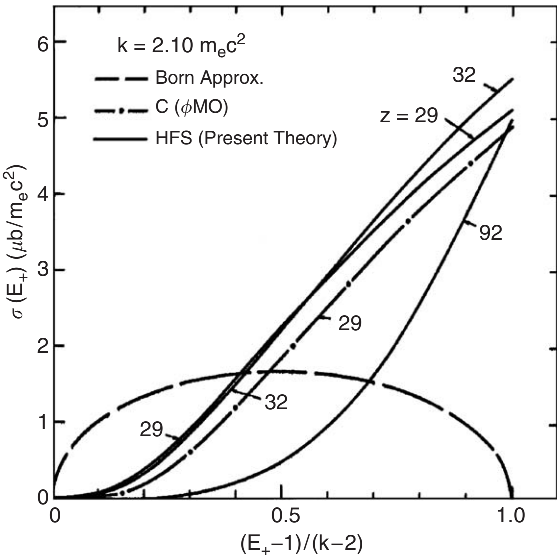

Fig. 1. Positron spectrum in pair production at $k = 2.10\,\mathrm{m_e c^2}$ of calculation in HFS field (solid line), compared with Born approximation results (broken line) and those of Overbo et al. (1978) (dotted–broken line) for point Coulomb field. Atomic numbers of target elements are attached to the curves. From Tseng and Pratt (1971).

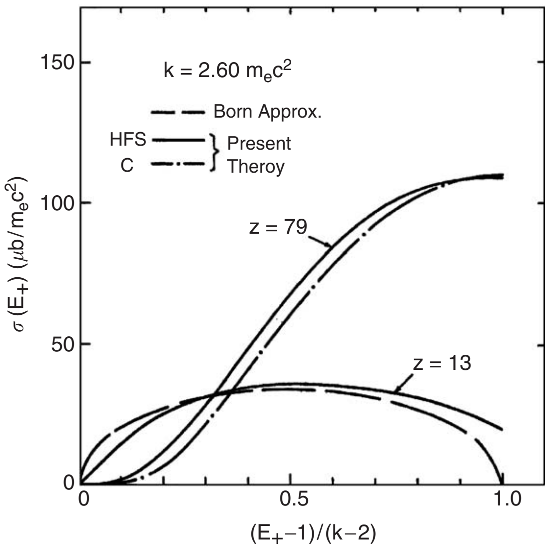

Fig. 2. Same as Fig. 1, except for $k = 2.60\,\mathrm{m_e c^2}$. From Tseng and Pratt (1971).

a Coulomb spectrum point to a screened spectrum point of shifted energy split, which led to what was called an energy shift theory (EST). This EST could also be derived from the behavior of screened and Coulomb wave functions at small distances, and their normalizations—changes in normalizations can be

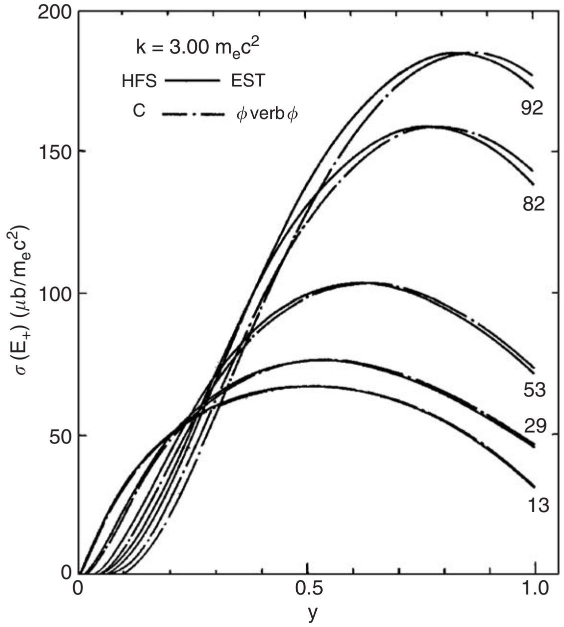

Fig. 3. Same as Fig. 1, except for $k = 3.00\,\mathrm{m_e c^2}$. From Tseng and Pratt (1972).

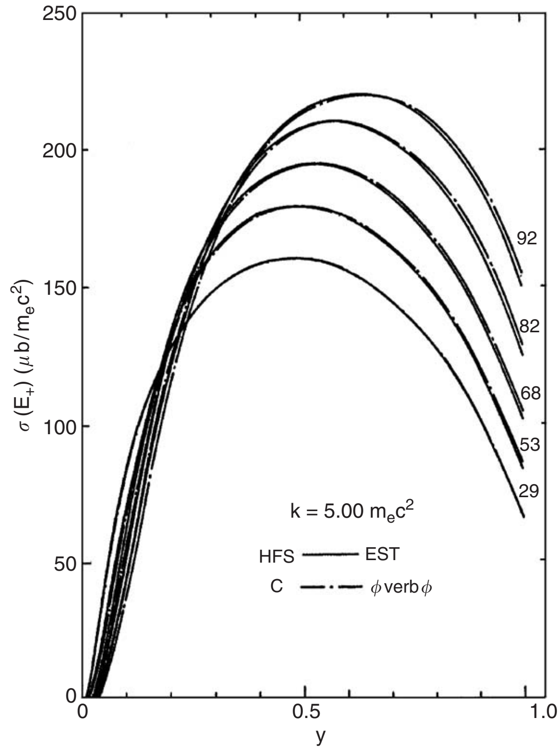

Fig. 4. Same as Fig. 1, except for $k = 5.00\,\mathrm{m_e c^2}$. From Tseng and Pratt (1972).

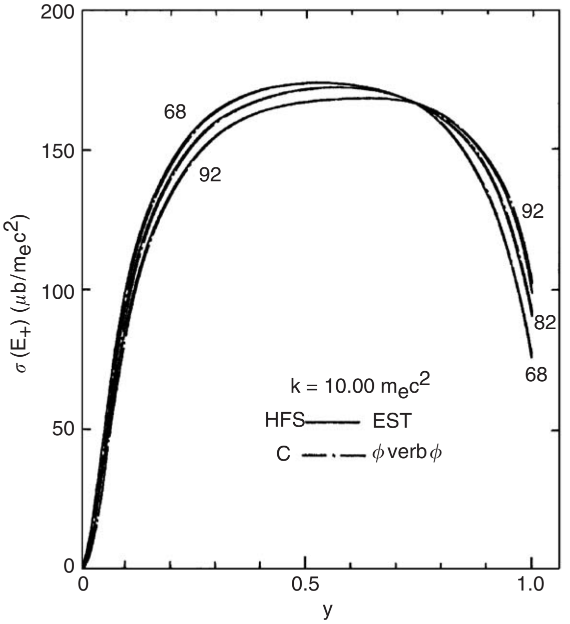

Fig. 5. Same as Fig. 1, except for $k = 10{,}00\,\mathrm{m_e c^2}$. From Tseng and Pratt (1972).

written in terms of energy shifts. (This still requires a regime determined by small distances, which is prior to the regime where form factor screening begins to matter.)

Finally, Tseng and Pratt (1974) studied the seven polarization correlations (between incident photon polarization and final positron spin) in pair production. An example is shown in Fig. 6. These correlations, which can be large, would permit the use of pair production as a polarizer of positrons, transmitter of polarization from photons to positrons, or analyzer of polarized photons. The correlations were found to be practically independent of atomic–electron screening, again indicating the validity of the normalization screening theory description.

Two later papers of Tseng and Pratt (1980, 1981), representing collaboration after Tseng had returned to Taiwan, focused on the near threshold regime, as low as $1.063\,\mathrm{MeV}(2.080\,\mathrm{m_e c^2})$,where new experimental results had become available. It was difficult to draw any clear conclusions from the limited experimental data. Some experimental results were consistently 30% above theory, as in two experiments of Avignone and his coworkers for $Z = 82$ and 92 at $2.190\,\mathrm{m_e c^2}$. But five different experiments for $Z = 92$ at $2.190\,\mathrm{m_e c^2}$ give a range from 30% above to 30% below theory. The experiment of Coquette for $Z = 32$, shown in Fig. 7, was striking in showing agreement with theory at lower and higher energies, but a 30% discrepancy in the intermediate energy region around $2.127\,\mathrm{m_e c^2}$.

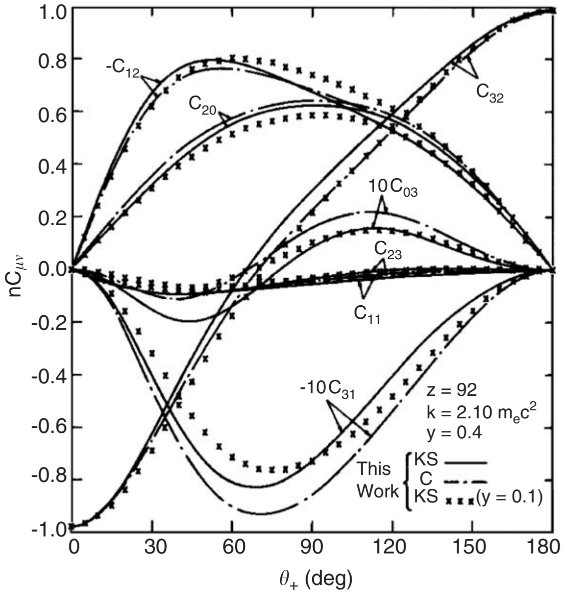

Fig. 6. Atomic-field pair-production correlations C_{ij} for the case $Z = 92$, $k = 2.10\,\mathrm{m_e c^2}$, energy split $y = 0.4$ and 0.1, with $y = (E_+ - 1)/(k - 2)$. KS and C refer to the Kohn–Sham and point-Coulomb potentials. From Tseng and Pratt (1974).

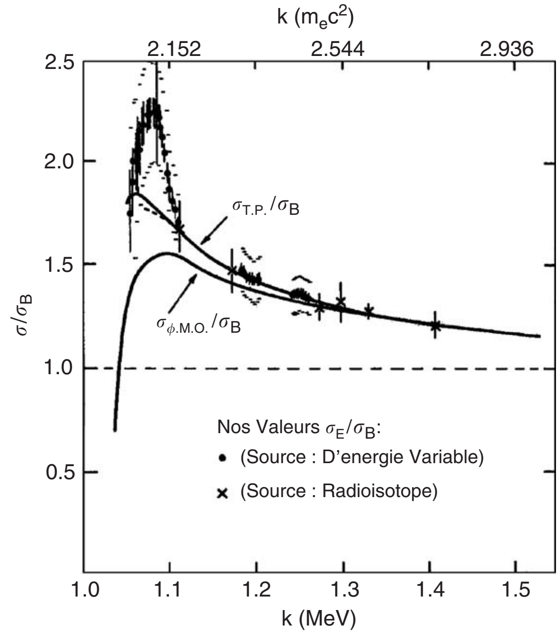

Fig. 7. Comparisons between theory (solid line) and the experiments of Coquette for the ratio σ/σ_B, where B refers to the point Coulomb Born approximation, TP to the work of Tseng and Pratt, and OMO to the work of Overbo, Mork and Olsen. The figure is superimposed on Fig. 1 of Coquette. From Tseng and Pratt (1981).

Unfortunately, except for the work of De Braeckeleer et al. (1992) discussed in Section 5, we are not aware of subsequent experimental work to resolve these problems.

Tseng and Pratt (1980) also used their numerical calculations, to verify that in this regime there was not high sensitivity to the choice of atomic model, and to conclude that the EST (energy shift screening theory) could still be used, with a suitable choice of energy shift.

4. Later work, 1990–1997

In the 1990s Tseng (1990, 1994a, b, 1995a, b, c, 1997) published a substantial amount of additional work, extending the results which he had previously presented.

In two papers Tseng used partial wave and multipole sampling techniques, originally developed for bremsstrahlung calculations, to extend numerical results for the pair production to higher energies. Tseng (1990) reported a calculation for $Z = 82$ at 10 and $15\,m_ec^2$. Results in the point Coulomb potential agreed with those which had been obtained by Overbo et al. (1968). Screening effects were less than 2%, and they were no longer well estimated by the EST (energy-shift screening theory). Later Tseng (1994) obtained results for 10 MeV photons on Uranium. Screening now decreased the cross section in most regions of the positron spectrum which contributed to the total cross section, by as much as 9% in the region of low positron energies, and by 3.5% in the total cross section. EST was not good, but form factor screening theory was now good.

Two further papers studied the angular distribution (Tseng, 1995c) and polarization correlations (Tseng, 1997) at 5 and $10\,m_ec^2$ for $Z = 1$ and 92, also for the polarization correlations at $Z = 50$. (Studies of the spectrum at such energies had been reported previously.) Born approximation worked well for low Z, as one would expect, and for high Z it gave better results for angular distribution shapes than it had for the spectrum. The high energy approximation, which was not yet good for the spectrum, appeared to give much of the angular distribution shape. Both angular shapes and correlations were almost independent of screening at these intermediate energies.

Three other papers studied the low energy spectrum (Tseng, 1994a), angular distribution (Tseng, 1995a) and polarization correlations (Tseng, 1995b), for $k = 2.001$, 2.01 and $2.10\,m_ec^2$, for $Z = 1$, 6, 13, and 82. In this more systematic study of the spectrum near threshold Tseng (1994a) found that screening becomes large (with increasing Z, decreasing energy, and decreasing positron energy), and as it does so (increasing the cross section) the EST fails, while normalization screening generally works very well. In his study Tseng (1995a) found that angular distribution shapes were independent of screening, consistent with the validity of normalization screening theory. Born approximation shapes remained better than the corresponding Born approximation predictions for the spectrum. This led to the possibility of simple parameterizations of the shape, as in bremsstrahlung and photoeffect. Finally, Tseng (1995b) confirmed that polarization correlations at these energies are independent of screening, again confirming the normalization theory of screening.

5. . Other work on low-energy pair production

The anomaly in threshold pair production from Ge reported by Coquette (1977, 1978, 1979, 1980) was reexamined by De Braeckeleer et al. (1992), here denoted DAG. They reported both new experimental results (see Fig. 8) and further theoretical discussion. With substantially smaller error bars than in previous work, they continued to see an anomaly, particularly at 1082 keV, which was consistent with Coquette's value within the combined error bars, although substantially smaller (15% above standard theory, rather than some 25%).

De Braeckeleer et al. (1992) also suggested that, since in this regime final electron and positron kinetic energies are small (around 30 keV), the final state Coulomb interaction between them should be considered. Making a crude estimate of such a correction factor (with results

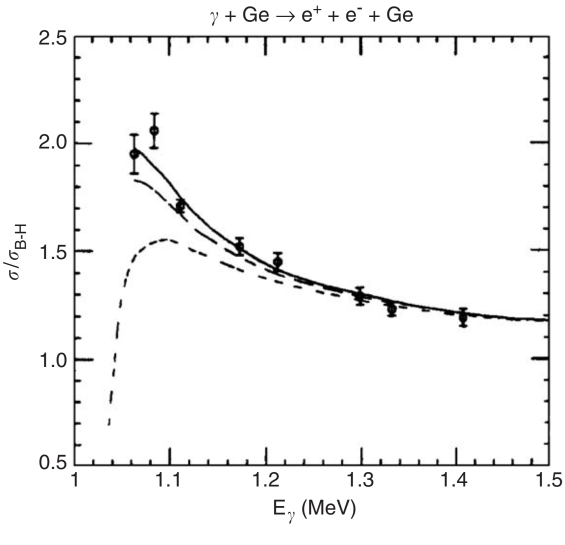

Fig. 8. Results of DAG for the pair production cross section in Ge in units of the Bethe–Heitler cross section. The error bars are relative, absolute cross sections were normalized to the predictions at 1836 keV. The solid curve shows DAG's calculation, including the electron-positron final state interaction; the dashed and dash–dotted curves show the standard theory, which neglects this interaction, for a screened and bare point nucleus, respectively. From De Braeckeleer et al. (1992).

shown in Fig. 8), DAG felt that they achieved qualitative agreement with their experiment. There was close agreement at 1063 and 1112 keV, with the experimental value at 1082 keV still 8% higher.

We are not aware of any further work on low energy pair production.

There has been at least one further use of low energy pair production data. Solberg et al. (1995) used pair production cross section data in a dispersion relation to determine the Coulomb and screening corrections to the Delbruck Born approximation forward scattering amplitude. However Carney and Pratt (1999) pointed out that the screening corrections used (based on screening corrections to the Born term and shifted Coulomb values) were not valid near the pair production threshold. Carney and Pratt (1999) redid the calculation, using low energy pair production data based on Tseng's work. (They also included contributions from the bound pair production process, which had incorrectly been omitted in the dispersion relation. The two contributions are of comparable magnitudes.)

6. Questions and opportunities

The main question regarding the experimental situation in low energy pair production is whether further and more accurate experiments can confirm the apparent disagreement with present theory. Is the anomaly in Ge at 1082 keV real, and what is its magnitude? Is the indication from some other experiments (but not all) that low energy data in other elements is systematically higher than theory correct?

The main questions regarding the theory of low energy pair production concern the appropriateness of the use of independent particle approximation (IPA). As noted, IPA neglects the role of correlation and exchange effects, in the initial and final atomic states, and with the final electron/positron pair. Since inner shell electron binding energies are similar to the kinetic energies of the created electron and positron, this latter is a question. Final state correlations also include the interaction between the created electron and positron, as noted by DAG. When correlations are considered, there are additional final states, involving excitation or ionization of the target, which must be included, since the experiments are inclusive rather than exclusive. (Within IPA they are included at the level of a sum rule approximation.) It does not appear that there should be any significant contribution from the bound pair production process, unlike in the Rayleigh and Compton scattering processes, discussed by Pratt et al. (1994).

Other theoretical issues would include the consideration of radiative corrections, the truncations made in multipole and partial wave series, and numerical questions regarding the accuracy of solution of differential equations for wave functions and integrals for matrix elements. However these matters seem unlikely to be of concern in this low energy region.

The questions we have raised above also represent opportunities for obtaining improved understanding of the role and behavior of correlations, and the use of sum rules for inclusive processes. More differential studies could evidently help clarify the behavior of these mechanisms. One could also imagine the use of COLTRIMS techniques to identify the ionization component of the process and determine its magnitude at low energy. (At high energy the ionization component is small, but it none-the-less become the dominant mode for photon ionization of atoms.) Bound pair production has been of recent interest, and it has also been studied through the inverse process of one photon positron annihilation, as has been discussed by Bergstrom et al. (1996)

Acknowledgments

This work was supported in part by NSF grant 0201595.

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