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Delbrück contribution in the elastic scattering of 1.115-MeV photons

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Differential cross sections for the elastic scattering of 1.115-MeV photons from tungsten ($Z=74$) and lead ($Z=82$) have been measured at angles ranging from 30° to 135°, using a high purity coaxial germanium detector. The experimental results are compared with *S*-matrix theoretical calculations of Rayleigh scattering cross sections, which also include contributions arising from the nuclear Thomson amplitudes and the Delbrück amplitude in lowest order Born approximation. The present experimental data at 1.115 MeV indicates that Delbrück amplitudes calculated with lowest-order Born approximation, when combined with *S*-matrix Rayleigh scattering amplitudes, are sufficient, as has previously been observed at 1.332 MeV for a number of high-*Z* elements, and at 1.121 MeV and 1.173 MeV for $Z=92$. This result for $Z=74$ and $Z=82$ at 1.115 MeV provides further confirmation that the Delbrück amplitudes calculated with lowest-order Born approximation are sufficient for energies at and below 1.332 MeV, in contrast to the situation at 2.754 MeV where Coulomb corrections to the Delbrück amplitudes are significant for high-*Z* elements.

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INTRODUCTION

Elastic scattering of photons is an important photon-atom scattering process in which a photon can be scattered through four different mechanisms, namely (i) Rayleigh scattering by the bound atomic electrons, (ii) Delbrück scattering by electron-positron pairs virtually created by the static Coulomb field surrounding the nucleus, (iii) nuclear Thomson scattering by the nuclear charge distribution, and (iv) nuclear resonance scattering by the giant dipole resonance. The process is called elastic because no energy is transferred to the internal degrees of freedom of the atom, which remain unchanged, and no additional photons are radiated. The individual contributions of these elastic scattering mechanisms are dependent on incident photon energy, the atomic number (Z) of the target atom and the angle (θ) of scattering. The experimentally measurable physical quantity is the differential scattering cross section in which a beam of photons is incident on a target and the scattered photons (scattered by any of the mechanisms mentioned above, which are not physically distinguishable) in a particular direction are detected with a suitable detector. A detailed discussion of the Rayleigh, Delbrück, and nuclear amplitudes, focusing in particular on the soft gamma ray regime (59.5 keV to 1.33 MeV), where all the amplitudes interfere considerably, has been given by Kane *et al.* [1]. There have been many experimental efforts during the past four decades to measure the elastic differential scattering cross sections for photon energies around 1 MeV [2].

In the few MeV range and below the nuclear amplitudes are primarily due to scattering off the nuclear charge distribution, allowing a simple treatment as scattering off a single free particle of charge Ze (the nuclear Thomson amplitude). Whereas for the Rayleigh amplitudes calculations in the *S*-matrix formalism using a partial wave expansion in a self-consistent central potential have long been available [3,4], the corresponding *S*-matrix description of the Delbrück amplitude has not been realized, though a formal treatment [5] and some limited numerical results [6] have been reported. Therefore, in the few MeV range the Delbrück amplitudes are generally treated in lowest-order Born approximation [7]. An adequate theoretical treatment of higher order effects (Coulomb corrections) for all angles in this regime has to date not been achieved. Results do exist for forward angle based on the optical theorem [8,9], but it is not clear how to extend these to finite angles, which are amenable to experiment. Other treatments beyond lowest order Born approximation are generally restricted to small angles or higher energies. Comprehensive discussions of the history and status of Delbrück scattering have been given by Papatzacos and Mork [10], Milstein and Schumacher [11], and most recently by Schumacher [12]. A major collection of numerical data for the Delbrück amplitudes was given by Falkenberg *et al.* [13]. This was used by Hubbell and Bergstrom [14] in their comparison of the Delbrück contribution to scattering with that of other photon atom processes. The paper [14] also includes an extensive bibliography.

Experimental results for elastic scattering on high-*Z* elements in the soft gamma ray regime allow a consideration of (1) situations where the Delbrück scattering amplitudes are

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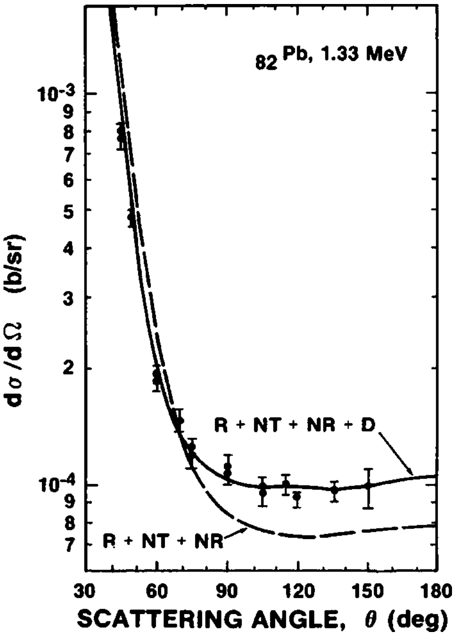

FIG. 1. Figure 4.1 taken from Kane *et al.* [1], showing comparisons of experiment (circles with error bars). The theoretical predictions include the Rayleigh, nuclear Thompson, and lowest order Born Delbrück amplitudes (solid line) and also the predictions with Delbrück omitted (dashed line). Results are for elastic scattering from ^{82}Pb at 1.33 MeV at a wide range of scattering angles.

required, in addition to the Rayleigh amplitudes, to obtain agreement within the experimental error and (2) in the case that the Delbrück amplitudes are important, whether lowest order Born approximation is sufficient. Researchers at the University of Gottingen started a series of experiments at 2.754 MeV, where Delbrück amplitudes dominate, from 1975 onwards [15–22], the contribution of Rayleigh amplitudes being minor. Basavaraju *et al.* [23], and Muckenheim and Schumacher [24] performed experiments at 1.332 MeV and found need for the inclusion of Delbrück amplitudes, but lowest order Born approximation was sufficient. This is seen in Fig. 1 (Fig. 4.1 of Ref. [1]), which compares experimental results for the elastic scattering of 1.332 MeV photons by lead with theoretical results obtained with and without the (lowest order Born) Delbruck amplitudes included. In contrast, experimental results at 2.754 MeV for Pb, Bi, TH, and U deviate from cross sections which utilize the lowest order Delbrück amplitudes by factors as large as ~2, through smaller for larger angles, as seen in Fig. 2 (Fig. 4 of Ref. [12]) for uranium ($Z=92$). As is discussed in [12] the discrepancies at 2.754 MeV for the different Z considered can be empirically corrected by a first Coulomb correction term to the amplitude of relative order $(Z\alpha)^2$ (as has also been observed at 9 MeV [12]). At 1.332 MeV where there are also fairly extensive measurements for different Z the lowest order Born result appears to be sufficient. However, the evidence that this is also the case at still lower energies is more limited. There are measurements at 1.121 MeV and 1.173 MeV confirming this, but only for $Z=92$ [25].

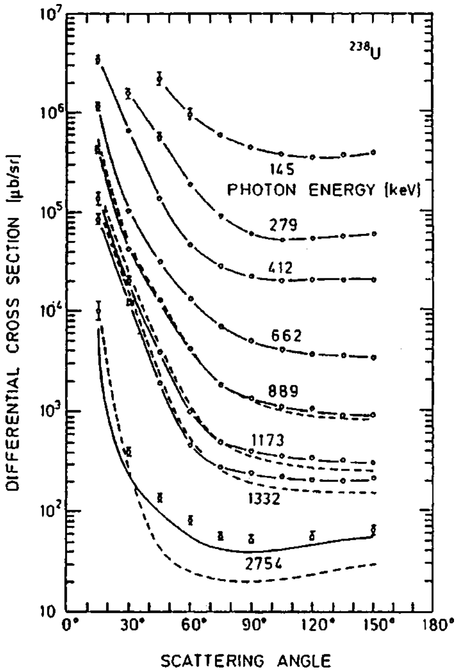

FIG. 2. Figure 4 taken from Schumacher [12], showing comparisons of experiment (circles with error bars). The theoretical predictions include the Rayleigh, nuclear Thompson, and lowest order Born Delbrück amplitudes (solid line) and also the predictions with Delbrück omitted (dashed line). Results are for elastic scattering from ^{238}U for energies ranging from 145 keV to 2.754 MeV at a wide range of scattering angles.

The motivation of the present experiment, with a HPGe detector and improved computer assisted data acquisition and analysis systems, using a monochromatic (^{65}Zn, 1.115 MeV) photon source with high-Z target atoms (Pb and W), is to further establish (for Z other than 92) whether we can rely on lowest order Born approximation for Delbrück scattering amplitudes at energies lower than 1.332 MeV. A somewhat similar situation has been observed [26,27] in the energy dependence of photoeffect cross sections, where the Born approximation energy dependence is corrected, in addition to a Stobbe factor, by further terms (but in $Z\alpha$). The terms are small in the energy range 0.5 to 2.0 MeV but they become large at higher energy (3 MeV and above), in high-Z elements decreasing the cross sections by a factor of 2.

EXPERIMENTAL METHODS

A 200 mCi ^{65}Zn source (with a half-life of 243.8 days), procured from the Bhaba Atomic Research Center, Mumbai, India, was used as source of 1.115-MeV mono-energetic gamma ray photons. The source was encapsulated in a stainless steel capsule of dimension 1.0 cm diameter and 1.1 cm length which was enclosed in a cylindrical block of lead (11.0 cm diameter and 15.32 cm length). Solid tungsten and lead scatterers in the form of square (5 cm × 5 cm) sheets and of thicknesses of $1.93-12.35\ \mathrm{g/cm^2}$ and $1.135-3.80\ \mathrm{g/cm^2}$, respectively, were used. It is important that the target materials be thick enough to have a sufficient number of atomic targets yet be thin enough (much less than the mean free path of the photon) to avoid multiple scattering

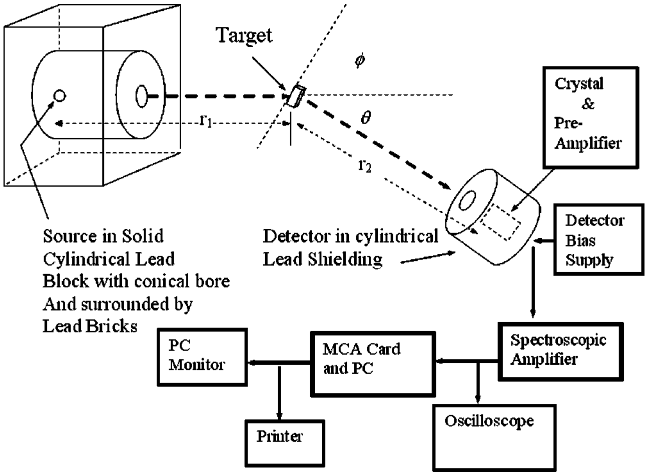

FIG. 3. Schematic diagram of the experimental arrangement.

in the target. The source to scatterer target distance r_1 and target to detector distance r_2 varied from 44.8 cm to 72.5 cm, and from 23.0 cm to 61.4 cm, respectively. The angular spread in the reflection geometry is least when $\sin\Phi/\sin(\theta-\Phi)=r_1/r_2$, where ϕ is the angle between the incident photon beam direction and the target axis and θ is the mean angle of scattering. For 30° and 60° both reflection and transmission geometry were used. For larger angles 90°, 120°, and 135° reflection geometry was used. The maximum angular spread of the scattering angle was 3°. A schematic diagram of the experimental arrangement is shown in Fig. 3.

A coaxial high-purity germanium detector (Oxford Instruments) was placed in a lead shielding with a hole of diameter 10.2 mm and length 44.3 mm. The relative efficiency of the detector as measured by the manufacturer was 39.8%. The full width at half maximum (FHWM) at 1.33 MeV was 1.77 keV and the peak to Compton ratio was 67.1. The scatterer targets were fixed in a perspex holder and placed in the center of a well-graduated (angle) circular wooden tabletop. The center of the target was aligned coaxially with the source collimator and the center of the detector face. The detector assembly was mounted on a moveable cart to align the detector at different scattering angles as well as for changing the distance between the scatterer and the detector.

The background spectrum was acquired by removing the target from the perspex holder for a considerable length of time before and after each scattered spectrum was recorded. The background spectrum was loaded first, normalized and subtracted from the scattered spectrum. The experimental differential scattering cross sections were determined by using the following relation:

$$\frac{d\sigma}{d\Omega}=\frac{1}{N_{\mathrm{AT}}T}\left[\frac{r_1^2}{\exp(-\mu_1^{\mathrm{air}}r_1)}\right]\frac{s_{\mathrm{ref}}}{S_{\mathrm{strong}}}\frac{N_{\mathrm{scatt}}}{N_{\mathrm{ref}}},$$

where N_{scatt} is the net scattered counts per unit time, N_{ref} is the number of counts from a weak reference source (of 1.115-MeV photons) placed at the position of the target with the target removed, r_1 is the source to target distance, N_{AT} is the number of atoms in the target, μ_1^{air} is the attenuation coefficient of air, and s_{ref} and S_{strong} are the intensities of the weak reference source and the main experimental source, respectively. The ratio $s_{\mathrm{ref}}/S_{\mathrm{strong}}$ was measured by placing the two sources in the same cylindrical lead collimator and placing two lead bricks in front of the opening mouth of the collimator and counts were recorded at different distances the detector was placed in line with beam. The ratio was also found by counting the photons from the two sources placed at a long distance away without any shielding or collimation of the sources and the detector. The geometry of the transmission factor T is depicted in Fig. 4, and it is given by

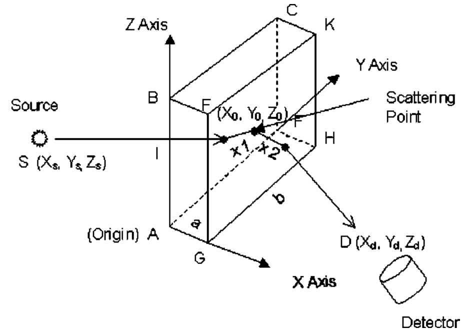

FIG. 4. Depiction of the transmission factor (T) geometry.

$$T\equiv\frac{1}{V}\int_{\mathrm{scat}}dV\exp(-\mu_1x_1)\exp(-\mu_2x_2).$$

where x_1 and x_2 are the distances traversed in the target before and after the scattering event, respectively, and μ_1 and μ_2 are the corresponding linear attenuation values for those respective path lengths, which are equal for elastic scattering, i.e., $\mu_1=\mu_2$. Determining the transmission factor required integrating over all possible scattering paths in the target. The attenuation coefficients of tungsten and lead required in the determination of the transmission factor T for 1.115-MeV photons are found through direct measurement to be 0.05979(5) ± and 0.06013(6) in cm^2/g, respectively. The corresponding theoretical data taken from Hubbell and Seltzer [28] are 0.06026 and 0.06054, respectively, and are within 1.2% of the measured values.

It is very important to minimize the ambient background signal as far as is possible because elastic scattering at 1.115 MeV is a low event photon-atom interaction process. The errors associated with possible background radiation were minimized by performing the experiment in a large room and shielding the source and detector assembly suitably by lead bricks. Extensive tests were made altering the geometry of the shielding of the source and the detector relative to the floor and walls of the room in which the experiment was carried out to minimize the background counts. In the present experiment the effects of Compton scattering, pair production and bremsstrahlung are not important, as they do not contribute to the elastic peak for the scattering angles at which measurements were made with this high photon energy. The experimental run for each target continued for more than hundred hours at a time to obtain good statistics. To minimize the pulse pileup effects due to longer

TABLE I. Differential scattering cross sections for tungsten, $Z=74$, for 1.115-MeV photons at a range of scattering angles. Predictions for the cross section are given based on the Rayleigh and nuclear Thompson amplitudes only (R+T), and also including the Delbrück amplitude in lowest order Born approximation (R+T+D). The experimental results are given with error in column 4 and the deviation from the best theoretical prediction is given in column 5.

Scattering angle	R+T (mb/sr)	R+T+D (mb/sr)	Experiment (mb/sr)	% Deviation from R+T+D
30°	19.3	17.5	17.8(6)	+1.71
60°	0.701	0.616	0.601(23)	−2.43
90°	0.210	0.224	0.218(9)	−2.68
120°	0.163	0.186	0.191(9)	+2.68
135°	0.158	0.182	0.190(11)	+4.39

peaking time proper pole-zero adjustment was made using a high quality spectroscopic amplifier by setting the peaking time to bring the trailing edge of the amplifier output pulse to baseline with minimum overshoot or undershoot by observing sample output connected to an oscilloscope. The electronic drift and specially the gain stability of the signal amplification chain was checked by recording direct spectra acquired with a ^{65}Zn weak source at different long intervals of time from time to time and by checking for any channel drift of the photopeak.

In addition to the automatic channel by channel background subtraction, the subtraction was also done manually by calculating the total counts under both the scattered photopeak spectrum and the corresponding background spectrum (recorded before and after the scattered spectrum to account for any channel drift). The difference in net counts was never found to exceed 1.8%. Decay correction of the source is negligible since the half-life of ^{65}Zn is 243.8 days corresponding to a reduction in the number of emitted photons by a factor of only 0.997 after 100 *h*. The target materials were specified to be 99.9995% pure (Alfa Aeser). The overall accuracy in the present experimental data is within 5%.

DISCUSSION

The present experimental results are given in Tables I and II, and they are represented graphically in Figs. 5 and 6, for tungsten and lead targets, respectively. Comparisons are made with theoretical values for the elastic scattering cross section both neglecting the Delbrück amplitude (i.e., using the Rayleigh and nuclear Thompson amplitudes only) and also including the Delbrück amplitude calculated in the lowest order Born approximation. The Rayleigh amplitudes are calculated in the second order *S*-matrix formalism in a self consistent Dirac-Slater type central potential [3,4] and are expected to be accurate at the one percent level. We observe that at 1.115 MeV, as with previous results at 1.332 MeV and for the limited available results at 1.121 and 1.173 MeV, the Delbrück amplitude needs to be included for agreement with experiment, but the result for the Delbrück amplitude in lowest order Born approximation appears to be sufficient given the experimental error of ~5%. The elastic scattering cross section calculated without including the Delbrück amplitude is clearly seen to underestimate the experimental results at large scattering angles. It is not surprising that need

TABLE II. Differential scattering cross sections for lead, $Z=82$, for 1.115-MeV photons. Comparisons of experiment with theoretical predictions are as in Table I.

Scattering angle	R+T (mb/sr)	R+T+D (mb/sr)	Experiment (mb/sr)	% Deviation from R+T+D
30°	13.9	12.6	12.9(4)	+2.38
60°	0.349	0.319	0.316(14)	−0.94
90°	0.100	0.108	0.104(5)	−3.7
120°	0.0813	0.0946	0.0976(26)	+3.17
135°	0.0809	0.0950	0.0982(34)	+3.36

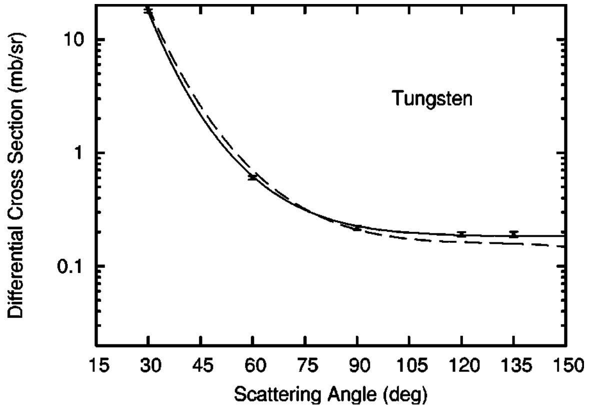

FIG. 5. Differential cross sections for elastic photon scattering from tungsten ($Z=74$) at 1.115 MeV, comparing experiment (circles with error bars) with the theoretical predictions including the Rayleigh, nuclear Thompson, and lowest order Born Delbrück amplitudes (solid line) and also the predictions with Delbrück omitted (dashed line).

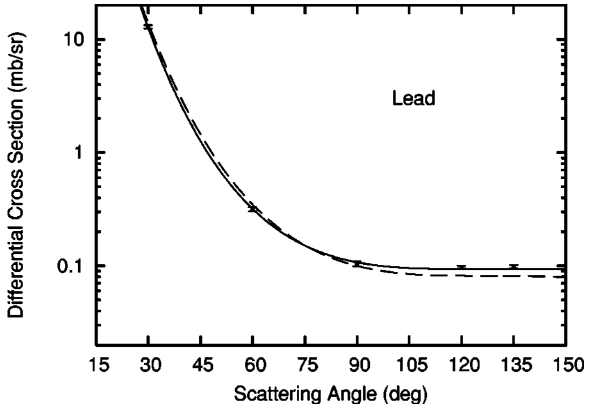

FIG. 6. Differential cross sections for elastic photon scattering from lead ($Z=82$) at 1.115 MeV, comparing experiment (circles with error bars) with the theoretical predictions including the Rayleigh, nuclear Thompson, and lowest order Born Delbrück amplitudes (solid line) and also the predictions with Delbrück omitted (dashed line).

for the Delbrück scattering amplitude is seen at this energy (1.115 MeV) as previous experimental results for scattering from uranium for a wider range of energies (Fig 2) show a need for inclusion of the Delbrück amplitude at energies as low as 889 keV. Referring to the last column in Tables I and II there is evidence of a common behavior in the deviation of the experimental results from the best theoretical results, with a positive deviation at the smallest angle and larger angles, and a negative deviation in between. However, the deviations in all cases are at the few percent level, and no definite conclusion can be made given the experimental error of ~5%, other than to note the similar behavior for both *Z*. Greater experimental precision may reveal whether this indicates, for example, a modest Coulomb effect at this energy. In this experiment there is no evidence for any deviation from the Born Delbrück amplitude greater than ~20%, to which the experiment would be sensitive.

In summary, results have been presented for elastic scattering from lead and tungsten at 1.115 MeV for a range of scattering angles. The conclusion from a comparison of these results with the theoretical predictions is that Delbrück scattering is significant and it is sufficiently well described within lowest order Born approximation at this energy (given the experimental uncertainty of ~5%), as was also the case for previous experiments at 1.332 MeV and for limited results at 1.121 and 1.173 MeV. These results therefore provide further evidence that the lowest order Born approximation for the Delbrück amplitude is sufficient for energies of 1.332 MeV and below. This in turn suggests that the Coulomb corrections to the Delbrück amplitude, known to be important at 2.754 MeV but not at 1.332 MeV, continue to be unimportant at still lower energies, as opposed to, for example, becoming more important at lower energies with 1.332 MeV representing a local minimum in the correction to the scattering cross section (as due to a sign change at the level of the amplitude).

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